Simulation of Hand Anatomy Using Medical Imaging: Supplementary Material

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1 Improved initial guess for the muscle tet mesh in example poses

We use the following strategy to stably deform the tet mesh into a pose that is sufficiently similar to the MRI pose. We first create a muscle fiber field on the muscle neutral tet mesh by solving a Laplace equation [Saito et al., 2015]. Then, we define

$$R \operatorname{diag}(a, 1, 1)\ R^T$$

as the plastic strain of each muscle tetrahedron, where R is the local frame defined by the muscle fiber direction, and a is the muscle contraction parameter [Saito et al., 2015]. We consider a simple optimization problem

$$\arg\min_{\mathbf{a},\mathbf{x}} \|\mathbf{L}\mathbf{a}\|^2 + c_1 E_{\text{elastic}}(\mathbf{a}, \mathbf{x}) + c_2 E_{\text{att}}(\mathbf{x}), \tag{1}$$

where $\mathbf{a}$ is the muscle activation at each tet, $\mathbf{x}$ contains the vertex positions of the muscle's tet mesh (our output, i.e., the initial guess), E_{elastic} is the elastic energy under the neutral rest shape, and E_{att} is the attachment energy of the muscle to the bones, modeled by zero rest-length springs at the attachment sites. Intuitively, this objective function attempts to discover the muscle activation so that, in the static equilibrium under the contraction, the attachment energy to the bones is small. This problem is a simplified version of the optimization problem defined in [Wang et al., 2021], because (1) it uses a much smaller space for the plastic strains, (2) it does not attempt to match the sparse markers, and (3) we treat the static equilibrium constraint as a penalty term in the objective function. The unconstrained optimization problem can be solved by using Newton's method. The DOFs provided by the contraction $\mathbf{a}$ are necessary, because the shape of a straight muscle would bend, as opposed to contracting, when two insertions come closer to each other. This is due to volume preservation terms in the elastic energy.

To simplify the problem, we performed model reduction over the vector $\mathbf{a}$. We manually select a few tetrahedra (usually 3-4) on the tetrahedral mesh and treat them as "handles," meaning that the contractions of these tetrahedra will control the entire $\mathbf{a}$. Then, we generate the subspace

basis $\mathbf{U}$ by calculating bounded bi-harmonic weights [Jacobson et al., 2011]. Then, the contraction $\mathbf{a} = \mathbf{U}\hat{\mathbf{a}}$, where $\hat{\mathbf{a}}$ is the subspace quantity, and our optimization problem becomes

$$\arg\min_{\hat{\mathbf{a}},\mathbf{x}} \|\mathbf{L}\mathbf{U}\hat{\mathbf{a}}\|^2 + c_1 E_{\text{elastic}}(\mathbf{U}\hat{\mathbf{a}}, \mathbf{x}) + c_2 E_{\text{att}}(\mathbf{x}). \tag{2}$$

It takes (on average) 2 minutes per muscle per pose to optimize Equation 2. Optimizing Equation 2 as opposed to Equation 1 gives us a 5× speedup and most importantly, easy optimization convergence when the dimension of $\hat{\mathbf{a}}$ is small.

2 Abandoned ideas for pose and spatial smoothness of volumetric plastic strains

2.1 Augmenting the method from [Wang et al., 2021]

The first abandoned idea was to augment the method described in [Wang et al., 2021] such that it supports pose-space smoothness across all example poses. The optimization is defined as follows,

$$\arg\min_{\mathbf{s_i},\mathbf{x_i}} \quad \sum_i^N \left(\|\mathbf{L}\,\mathbf{s_i}\|^2 + \alpha\mathcal{E}_{\text{MI}}(\mathbf{x_i}) + \beta\mathcal{E}_{\text{a}}(\mathbf{x_i})\right) + \gamma\|\mathbf{L_{ps}}\,\mathbf{s}\|^2, \tag{3}$$

$$\text{st.} \quad \mathbf{f}_{\text{e}}\big(\mathbf{F_p}(\mathbf{s_i}), \mathbf{x_i}\big) + \mathbf{f}_{\text{a}}(\mathbf{x_i}) = 0, \text{for each} \quad i = 1, 2, \ldots, N \tag{4}$$

where $\mathbf{L_{ps}}$ is a pose-space Laplacian matrix. Directly solving this is not feasible because the number of unknowns is enormous. Thus, we also use the block-gradient descent method, whereby in each iteration, we randomly select pose $k \in [1, N]$, and freeze unknowns related to all other poses. Then, the problem in each iteration becomes

$$\arg\min_{\mathbf{s_k},\mathbf{x_k}} \quad \|\mathbf{L}\,\mathbf{s_k}\|^2 + \alpha\mathcal{E}_{\text{MI}}(\mathbf{x_k}) + \beta\mathcal{E}_{\text{a}}(\mathbf{x_k}) + \gamma\|\mathbf{s_k} - \bar{\mathbf{s}}_\mathbf{k}\|^2, \tag{5}$$

$$\text{subject to:} \quad \mathbf{f}_{\text{e}}\big(\mathbf{F_p}(\mathbf{s_k}), \mathbf{x_k}\big) + \mathbf{f}_{\text{a}}(\mathbf{x_k}) = 0, \tag{6}$$

where $\bar{\mathbf{s}}_\mathbf{k} = \sum_{j\in N(k)} w_{kj}\mathbf{s_j}$ is constant during each iteration. By closely examining the objective function, we observed that this formulation has flaws. Remember that $\mathcal{E}_{\text{MI}}(\mathbf{x_k})$ and $\mathcal{E}_{\text{a}}(\mathbf{x_k})$ are terms to match the sparse observations. They penalize the distances of a sparse set of surface points to the target positions. On the other hand, the term $\|\mathbf{s_k} - \bar{\mathbf{s}}_\mathbf{k}\|^2$ is defined densely for each tetrahedron, and causes the shape of the volumetric mesh to match $\bar{\mathbf{s}}_\mathbf{k}$. To minimize the energy, in the region where there are no markers, the shape of the volumetric mesh follows $\bar{\mathbf{s}}_\mathbf{k}$. In the region where there are markers, however, these terms will combat each other. This is because in general the shape corresponding to $\bar{\mathbf{s}}_\mathbf{k}$ does not meet the markers. As a result, bumps appear when $\alpha, \beta > \gamma$. Conversely, the surface simply cannot meet the markers if $\alpha, \beta < \gamma$. We have implemented this approach and verified experimentally that the method suffers from the listed negative outcomes. To resolve this problem, we have to use a dense observation of the markers, which in turn causes a prohibitive time complexity. Therefore, this approach is not feasible and we abandoned it.

2.2 Solving using a geometric shape modeling method

To use a dense correspondence, namely a complete target surface, we could simplify our previous objective function and treat it as a pure geometric shape modeling problem. We define smoothness as the Laplacian of the deformation gradient [Sumner and Popović, 2004, Saito et al., 2015],

resulting in

$$\arg\min_{\mathbf{x_i}} \quad \sum_{i}^{N} \left(\|\mathbf{L}\,\mathbf{F}(\mathbf{x_i})\|^2 + \alpha \mathcal{E}_{\mathrm{MI}}(\mathbf{x_i}) \right) + \gamma \|\mathbf{L_{ps}}\,\mathbf{F}(\mathbf{x}))\|^2, \tag{7}$$

where $\mathbf{F}(\mathbf{x})$ is the deformation gradients of $\mathbf{x}$, and $\mathcal{E}_{\mathrm{MI}}(\mathbf{x_i})$ are now dense correspondences. Here, $\mathbf{F}(\mathbf{x}) = \mathbf{G}(\mathbf{x} - \bar{\mathbf{x}})$, where $\mathbf{G}$ is the gradient operator matrix and $\bar{\mathbf{x}}$ is the rest position of the volumetric mesh. As we can see, the deformation gradient $\mathbf{F}$ is a linear function of the position $\mathbf{x}$. The resulting $\mathbf{x}$ can then be converted to plastic strains. Because we are using isotropic hyperelastic materials, the plastic strain is essentially the symmetric matrix of the polar decomposition of the deformation gradient F for each tetrahedron. Although it is a quadratic energy and easy to optimize, this method cannot handle rotations well, leading to a large amount of inverted tetrahedra.

To resolve this problem, we attempted to add a non-linear inequality constraint that guarantees that the determinant F of each tetrahedron is positive, that is, $\det(F) > \epsilon$, where ϵ is a very small positive number such as 0.02. Nonetheless, this method creates extreme plastic strains whose determinants constantly hit the ϵ boundary, and whose eigenvalues span a wide range, such as from ϵ to 15.2. This causes extreme plastic strains when interpolating the plastic strains during simulation. Moreover, solving such a constrained nonlinear optimization is difficult. We have encountered failure muscles.

To address the problem of bad rotations, we also attempted to replace the Laplacian smoothing energy with a standard elastic energy [Smith et al., 2018]. By adjusting the resolution of the tetrahedral mesh, we were able to solve this optimization for every muscle. Nonetheless, this method still created extreme plastic strains. This is because an elastic energy penalizes the growth of the object, while the muscles are growing/shrinking all the time in different example poses. Therefore, we abandoned this method as well.

3 Tendon Groups

Table 1: **Tendon Groups.**

finger	palmar group	dorsal group
thumb	flexor pollicis longus	extensor pollicis longus
index	flexor digitorum superficialis, flexor digitorum profundus	extensor indicis, extensor digitorum
middle	flexor digitorum superficialis, flexor digitorum profundus	extensor digitorum
ring	flexor digitorum superficialis, flexor digitorum profundus	extensor digitorum
pinky	flexor digitorum superficialis, flexor digitorum profundus	extensor digiti minimi

In our paper (Section 5), we group tendons into tendon groups (shown here in Table 1). Groups contain either a single tendon or two tendons, as indicated. There is a separate group for the

palmar and dorsal side of each finger (10 groups total). On real hands, there are three tendons on the dorsal side of the thumb finger: extensor pollicis longus, extensor pollicis brevis, and abductor pollicis longus. The latter two are not visible in the MRI scan, and therefore we only simulate the extensor pollicis longus. The extensor digitorum tendon spans three fingers (index, middle, ring), and there is a tendon "bridge" (in the upper dorsal palmar area) connecting the three fingers; these bridges are not simulated. Similarly, we do not simulate the tendon "bridge" between the extensor digitorum and extensor digiti minimi.

References

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