Evaluating the Efficiency of Six-DoF Haptic Rendering-Based Virtual Assembly Training

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***Abstract*—Haptics plays an important role in training users to assemble mechanical components, such as airplane or car parts. Because mechanical components are often geometrically complex, efficient collision detection and six-DoF haptic rendering of contact are required for virtual assembly, and this has been extensively explored in prior work. However, as this article shows, this alone is not sufficient for efficient virtual assembly training. This article asks how to augment six-DoF haptic rendering of contact to maximize virtual assembly training efficiency, and proposes and measures several visual and haptic guidance strategies. Our visual strategies consist of displaying animations of the correct assembly path, motion indicator cues, and close-ups on difficult assembly path sections. Our haptic guidance consists of forces and torques that correct the trainee's deviation from the path. We investigate several haptic guidance strategies, including continuous forces and torques, force/torque nudging and anti-forces/torques. We designed a user study to evaluate the training efficiency of our proposed strategies quantitatively, using ANOVA and Tukey statistics. Our main finding is that the most efficient training approach is to use haptic rendering of contact in combination with visual animation-based guidance. Continuous forces, nudging, anti-forces and motion indicator cues were measured to be less effective.**

***Index Terms*—ANOVA, six-DoF haptic rendering, Tukey test, user study, virtual assembly and training.**

I. Introduction

Learning how to assemble complex machinery is a significant task in the manufacturing industry. Especially where the product components are complex, well-trained technicians are needed to perform the assembly during the original manufacturing process, and during maintenance. Virtual assembly training helps the users obtain prior knowledge of assembly tasks before performing them in the real world. In this way, the assembly personnel can be trained without access to the actual mechanical parts, and without the risks of damaging equipment or sensitive parts.

In this work, we focus on arguably the most commonly explored virtual assembly operation in literature, namely insertion/manipulation of a complex rigid object in a complex rigid environment. Our scenario, shown in Fig. 1, involves manipulating a rigid car starter motor into its place in the rigid engine, past the narrow clearances at the engine opening. We assume that the path planning problem *has already been solved,* by the engineers that designed these structures; using any method, i.e., path planning algorithms, haptic virtual design, manual trial and error, etc. Due to narrow clearances and non-intuitive translational and rotational motions required for successful assembly, it is not easy for trainees to learn and repeat the assembly trajectory. We study the task of *training* new users to perform the assembly, given a known (but complicated) insertion path in the six-dimensional rigid body configuration space. Unlike prior work which typically investigated simple geometric scenarios such as peg-in-hole insertions involving cylinders, tubes and cubes, we analyze virtual assembly of complex geometry typical of complex mechanical structures in engineering.

Complex structures bring specific challenges that do not manifest with simple geometry. Obviously, there is the stringent computational requirement of performing collision detection and computing stable contact forces at haptic rates (1,000 Hz). With recent advances in algorithms for haptic rendering, however, this task is now feasible, so we do not focus on the problem of haptic rendering in this paper. For haptic rendering, we use our previous work [1], [2], which accelerates collision detection using tree-organized points and signed distance fields, and stabilizes stiffness calculation regardless of the contact directions. Together, these methods enable stable haptic rendering of forces and torques at a resolution of 1 mm for both the car engine and starter motor geometries.

Even with the ability to render haptic forces and torques at such resolutions at haptic rates, significant challenges remain for virtual assembly training. The number of contacts is large, and the contact normals point in many directions. This makes the assembly difficult because the users cannot rely just on contact cues to understand how they should navigate the object past the obstacles. The collisions sites are often occluded and therefore efficient visualization of the contact geometry and the assembly path is very important. We designed and evaluated several visualization strategies, including displaying animations of the assembly path starting from the current configuration, displaying animation ghosts, using transparent rendering and automatically focusing the camera on key points along the assembly path. Virtual assembly can greatly benefit from haptics. In addition to the contact

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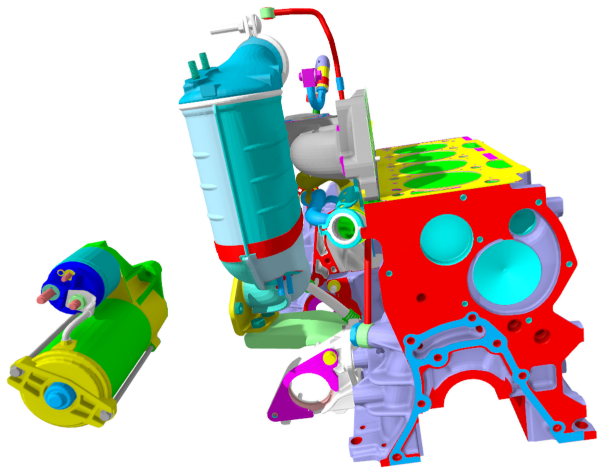
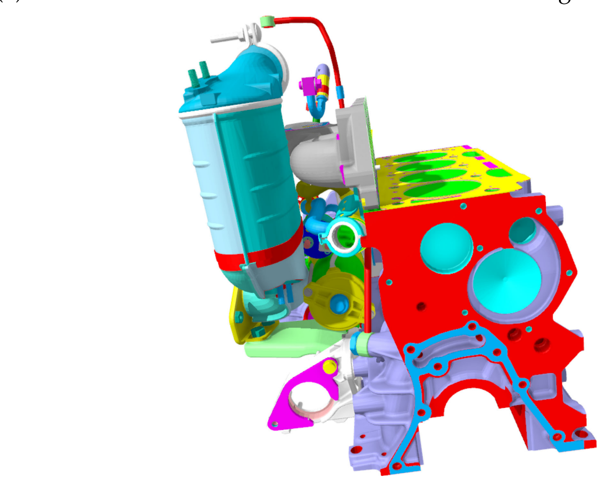


(a) Initial state. The starter motor is outside of the engine.



(b) Final state. The starter motor has been inserted into the engine.

Fig. 1. Virtual assembly of the car engine and starter motor.

forces, one can add haptic forces and torques that guide the trainee to the goal. We considered and evaluated 6D continuous guiding forces/torques, and short-time nudging forces and torques.

In our work, the virtual assembly path is six dimensional (translations and rotations) with many occlusions due to complex geometry. Our situation is different to, say, haptic-based motor skill learning, where the manipulation path is usually a simple 2D or 3D curve, consisting of straight lines or circle curves that are easily observable and understandable. Our assembly path is not just difficult to observe, it is also difficult to understand. The trainee needs to learn exactly how to navigate the starter motor past the obstacles at the entrance of the engine bay, and this skill requires good high-level understanding of the problem, as well as good hand coordination. The goal is therefore *not* to exactly repeat the known assembly path, but to understand and memorize the 6D assembly process. Therefore, a virtual assembly training system needs a good and proper mix of haptic contact forces, haptic guidance forces, and comprehensive and easy-to-understand visual guidance. Selecting this mix is a very non-trivial task because there are many strategies that one may potentially consider. In this work, we designed and evaluated 14 such training strategies. The objective of our work is not just to propose these strategies, but also to evaluate them quantitatively. Seven of the strategies were discarded due to their bad performance in our pilot study [3], or clearly suboptimal performance in our experiments. The remaining seven were evaluated using our user study on 56 subjects.

There has not been a prior work that has analyzed and evaluated (using a user study) the various training strategies for virtual assembly of complex geometry. Designing such a study is not an easy task. Unrelated factors must be eliminated so as to ensure the data reliability, and the results should be measured in an objective way. The study must be long enough to reliably measure the training success, but not too long or too boring for the user to lose focus and become impatient. Our study includes a training introduction, familiarization with haptics, a pre-training test, training with one of the 7 strategies and a post-training test. The entire process lasted 40-50 minutes for 1 subject; all subjects participated at different times and completely independently of one another. The familiarization section was designed to equip all users with the same prior knowledge before the real assembly task. We measured the assembly difficulty rate and assembly completion time in pre-training tests and post-training tests. We employ a pre-training test so that we can control for the fact that some people are innately better than others at assembly of 3D structures. We measure the efficiency of a strategy by statistically analyzing the differences between the pre- and post-training tests.

The results of our user study indicate that haptic contact feedback is very important for realistic virtual assembly. Our ANOVA and Tukey statistics [4], [5] measured (to p-values of 0.05 or less) that providing a proper amount of training guidance improves assembly performance. However, too much guidance impedes learning, due to passive participation. Most notably, the strategy of using continuous haptic guidance forces was measured to perform poorly due to passive participation. Visual guidance was measured to be better than haptic guidance at depicting complex six dimensional movements, to a significant statistical level (p-value under 0.05). Our quantitative results indicate that the recommended strategy for virtual assembly training is to employ haptic rendering of contact, but augment it with animations of the manipulated object progressing down the assembly path, launchable by the trainee whenever suitable or necessary during the training.

II. Related Work

Our goal is to design and evaluate the efficiency of several training strategies for virtual assembly in the presence of complex geometry. We therefore group the related work in two parts: virtual assembly, and training strategies.

A. Virtual Assembly

There are a number of research publications on virtual assembly design. Focusing on mechanical assembly with clearance fits, where the peg is sliding/rotating against the hole, Wang *et al.* proposed a novel force rendering model to obtain more realistic force feedback, by decomposing the assembly operation into three sub-procedures and then calculating the corresponding forces [6]. The virtual assembly system proposed by Sagardia *et al.* handled real-time collision detection, haptic force rendering, as well as proposed an interface for rigid body navigation for the real and practical assembly scenarios [7]. In addition to tactile and force feedback, Al-Ahmari *et al.* presented a fully functional virtual manufacturing assembly simulation system [8]. Gonzales *et al.* developed a physics-based modelling virtual reality system as a tool for training, design analysis and path planning in assembly [9].

In addition to building the system for virtual assembly, researchers investigated providing trainees with assembly cues. In general in haptics and virtual reality, such cues are referred to as *virtual fixtures*. In human-machine manipulation systems, virtual fixtures are commonly used to guide user's motion along the correct path. For example, they can provide surgical robot directional force feedback to ensure safe operation [10]. Virtual fixtures can assume the form of visual, haptic and audio feedback [11]–[13]. For virtual assembly, Christiand *et al.* presented a novel algorithm to determine the optimal assembly sequence of multiple components [14]. Simard *et al.* applied accurate haptic feedback forces for selecting topological entities (vertices, edges, faces) in 3D applications [15]. Tching *et al.* used two-step haptic guidance for virtual assembly: a geometry fixture to position the assembly objects, and kinematic constraints to perform the assembly task [16]. Blin *et al.* presented a novel haptic path planning algorithm, based on a RRT planner [17]. Hassan *et al.* developed a path planning optimization methods, based on Ant colony algorithm and potential field path planning concepts [18], [19]. Ladeveze *et al.* proposed a haptic assistance strategy by applying a path-constrained force to the user, once the path was found by a real-time path planner [20], [21]. Several publications proved in their user studies that offering realistic, multimodal feedback or guidance to the users benefits their performance. Yoon *et al.* showed that by providing the assembly sequence guidance, the subjects' performance on virtual assembly tasks has been improved [22]. Gallegos-Nieto *et al.* analyzed the influence of the virtual assembly training on the real assembly performance. Virtual assembly was proven to be effective in enhancing the participants' assembly skills, compared to those trained traditionally [23]. Results from Yin *et al.'s* user study indicated that vibrotactile and auditory feedback can significantly help to reduce the assembly completion time [24]. Similarly, Abidi *et al.* conducted a series of user studies and proved that VR-based training platforms with visual, auditory or force feedback cues are more effective than traditional training [25]. In our work, we investigate training techniques for virtual assembly of complex geometry, as commonly encountered in manufacturing practice. We research this problem comprehensively, by proposing and investigating 14 different training strategies. Several of these strategies were inspired by prior work in different haptic domains, but we are first to demonstrate how to apply them to haptic virtual assembly of complex geometry, and evaluate them in that context.

B. Training Strategies

Strategies to teach motors skills to disabled users are commonly investigated in haptic literature. Feygin *et al.* [11] investigated three training conditions (haptic; visual; haptic + visual) for teaching users to learn a 3D motion, and measured user's performance by asking them to reproduce the motion. For motor skill learning, Pareek *et al.* developed a haptic-based stroke rehabilitation system and compared how different levels of assistance affect performance improvements [26]. Researchers also presented a position error pattern to identify if the users are active or passive during learning [27]. For hand-writing, guiding forces were used to guide the user's hand along a pre-defined world-space trajectory, which has been proved to be an effective way of learning [28], [29]. Researchers also investigated more efficient methods to teach hand-writing. Park *et al.* examined the effectiveness of different haptic guidance modalities to children's handwriting skills acquisition [30]. Similar work was done in [31], [32].

The most commonly used guidance approaches employed in literature are constant guiding forces, progressive guiding forces, error amplification, and disturbance forces. Constant guiding forces guide the user along the path with constant force amplitude. Such a strategy, however, suffers from passive training [33]. The idea of the progressive guiding force approach is to decrease the force amplitude as the users progress in their training. Error amplification is another approach to improve the users' performance. It works by intentionally adding a learning difficulty, inspired from the ideas of over-training where a person improves his/her body's ability by strenuous exercise. Over-training has been applied and evaluated for visuo-motor transformations [34], wrist movement [35], and hand-writing learning [36]. The concept of haptic disturbance was first introduced by Lee *et al.* [37], as an extension of error amplification. In their work, they studied the efficiency of repulsive and noise-like disturbance forces on the motor learning process,and concluded that disturbance forces are helpful for the learning process. Several studies evaluated the concept of disturbance force. Lee *et al.* analyzed the effect of haptic disturbance on memorizing sequential selection tasks [38]. Lee *et al.* proposed and assessed a hybrid haptic assistance scheme, combining haptic guidance and noisy disturbance, for steering controlling learning [39].

Applying short force pulses ("nudging"; typically lasting 250 ms) was investigated in several publications on attention loss in brain injury recovery [40]–[42]. In these studies, the patients were asked to reach towards as many targets as possible in a fixed amount of time. The papers proved that haptic nudging is very good at keeping subject active. Therefore, the nudge force is a good attention acquisition strategy. However, it has so far not been adopted or evaluated as a training strategy for virtual assembly.

Teaching strategies are well-studied in the area of motor skill learning. However, these strategies cannot be directly applied to virtual assembly training. In motor skill learning, the strategies are designed to strengthen the subject's motor ability along very simple 2D paths, with little logical processing and without having to form 3D mental images of the involved geometry. In our system, the path is complicated,

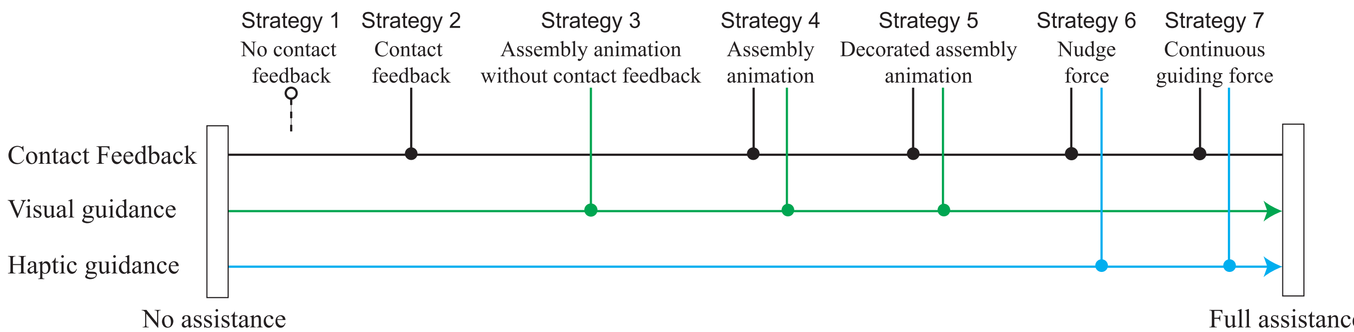


Fig. 2. Seven training strategies with different levels of training assistance.

including 3D translations and 3D rotations of complex geometry. Visual occlusions appear easily as the mechanical geometry moves in the environment. Furthermore, our learning does not only regard muscle memory, but also logical memory.

III. Training Strategies

We now discuss how to augment haptic rendering of contact with additional haptic and visual cues that help the trainees learn the assembly task. Assembly in the presence of complex geometry involves both translations and rotations, and these transformations are complex and not visually intuitive. We have designed fourteen different training strategies, and decided to keep seven of them for the user study (Fig. 2). Previous work [35], [37], [39] has established that the user will be largely passive if offered too much training assistance. However, if no training assistance is given at all, the users can quickly become confused. Much like generally in education, successful strategies therefore provide just the right amount of feedback, so that the user learns, but remains engaged and does not "slack off". Therefore, our user study (Section IV) does not merely evaluate the strategies, but also answers the question of how much training assistance is needed for efficient training. We now explain our training strategies. Everywhere in our work, we found that navigating the camera with the mouse is cumbersome for the trainees because they already are holding the haptic handle with one hand. It is much better if the trainee can manipulate the camera view directly with the haptic handle; we achieve this by mapping the camera viewpoint changes to the buttons on the haptic device.

A. No Guidance

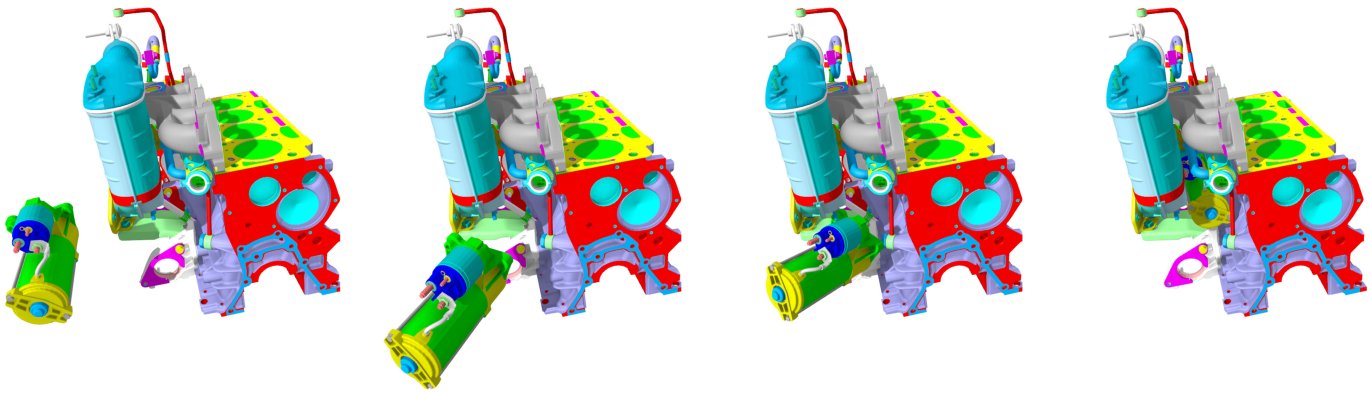
Our first two strategies serve as control strategies; neither of them provides any explicit training guidance. Our Strategy 1 ("**No contact feedback**") represents the bare-bone minimum where the system performs collision detection and computes the contact forces and torques, but renders zero forces and torques to the haptic device. The system only visually displays the current non-penetrating "God" object configuration, based on the current manipulandum configuration. In Strategy 2 ("**Contact feedback**"), we do everything as in Strategy 1, except that we also render the computed contact forces and torques to the haptic device. As expected, our user study (Section IV) demonstrates that Strategy 1 is poor, and that Strategy 2 outperforms it. This establishes that haptic rendering of contact is useful for virtual assembly. Both strategies were measured to be inferior to the strategies that involve training guidance, which we describe next. With the exception of Strategy 3, these strategies involve everything that Strategy 2 involves, plus they add various forms of training guidance.

B. Visual Guidance

Our system is able to provide visual guidance to the user in real time. It does so by showing an animation of the assembly path, starting from the user's current configuration to the goal. The animation is shown under properly chosen camera views for good visibility. Specifically, we designed the following three strategies. In Strategy 3 ("**Assembly animation without contact feedback**") and Strategy 4 ("**Assembly animation**"), when requested by the user by pressing a key on the keyboard, the system displays an assembly animation, starting from the user's current configuration, to the closest configuration on the path, and finally to the assembly goal (Fig. 3). In these two strategies, we show the assembly animation from the user's current camera view. The difference between Strategy 3 and 4 is that the former does not render the contact haptic forces and torques to the trainee, similar to Strategy 1.

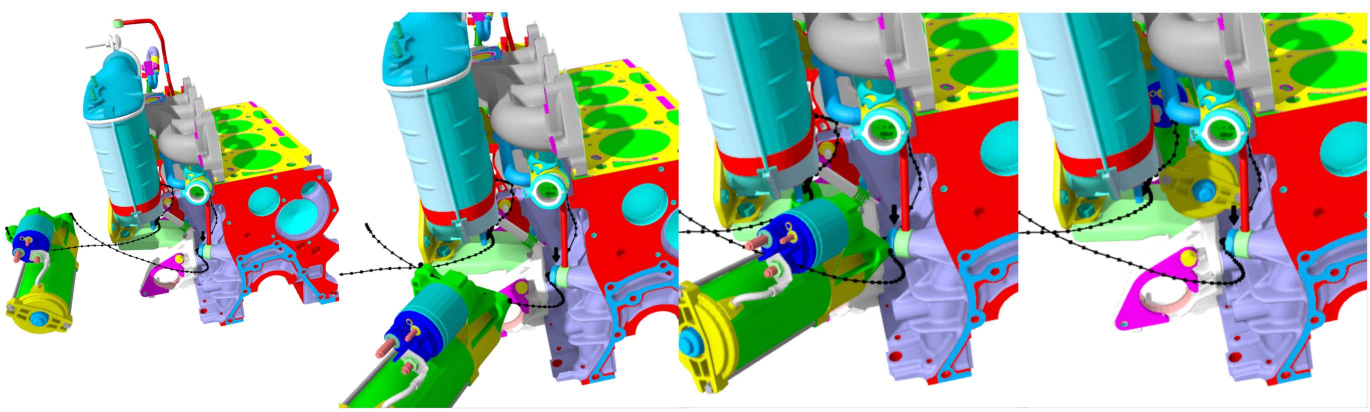
In Strategy 5 ("**Decorated assembly animation**"), we add additional visual assistance to the assembly animation. In our preliminary user study [3], we observed that the trainees were mostly stuck at the narrow clearance, where there are two screws and one protrusion on the engine block, and two protrusions on the starter motor, as indicated in Fig. 4. The key to solving the problem is to teach the trainees how to control the starter motor to bypass the protrusions on the engine. However, the solution path is easily occluded by the other parts of the engine. Therefore, we utilize the following additional training assistance in Strategy 5 (Fig. 4):

- Camera transformations. While showing the assembly animation, we smoothly transform the trainee's camera view to one that enables the trainee to have a much direct observation.
- Emphasize protrusions of the starter motor. The protrusions are indicated by two black spheres and their optimal trajectories are indicated by arcs.
- Emphasize protrusions of the engine. We use two flashing black arrows to delineate the protrusions.



(a) (b) (c) (d)

Fig. 3. Assembly animation training strategy (insertion of a car starter motor into the engine).



(a) (b) (c) (d)

Fig. 4. Decorated assembly animation training strategy (insertion of a car starter motor into the engine).

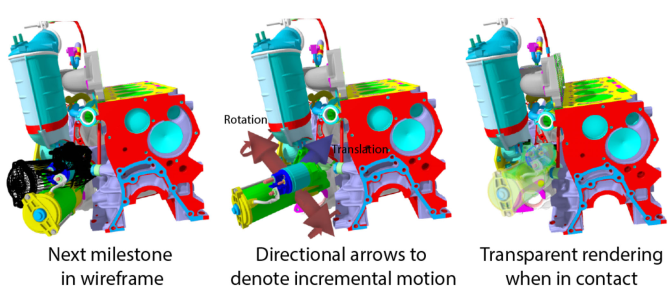


Fig. 5. Discarded visualization strategies.

We also tried three additional visualization approaches (Fig. 5), but discarded them due to poor performance: (1) automatically generate a set of equidistant representative milestones down the animation path, and always display the next milestone, in wireframe; (2) arrows indicating how the object should be incrementally translated and rotated to proceed down the assembly path; and (3) transparent rendering when the manipulated object is in contact. Approach (1) was causing substantial clutter on the screen and was not intuitive at all; rendering in solid transparent mode as opposed to wireframe was even worse. A similar problem occurred with the approach (2). The arrows caused too much clutter and were too difficult to interpret for the trainees, especially the arrows indicating the incremental rotation. Namely, when near a choke point on the path, the arrow direction changed too much and in seemingly unpredictable ways for the trainee to be able to use it effectively. The idea behind transparent rendering (3) is to show the contact sites which are otherwise blocked by the manipulated object. The object is rendered normally when not in contact, but switches to transparent rendering whenever in contact, with the contact sites displayed in red. The geometry visible through the transparent manipulated object caused great confusion; trainees often could not tell whether they are looking at the manipulated object, or the obstacles behind it. The flickering between normal rendering and transparent rendering also caused confusion. Hence, we discarded these strategies.

C. Haptic Guidance

We now describe our three haptic guiding strategies: continuous forces, nudging forces, and anti-forces. We assume that the engineers who designed the structure already properly completed their job, namely they determined that the assembly is possible, and produced a known assembly trajectory. We

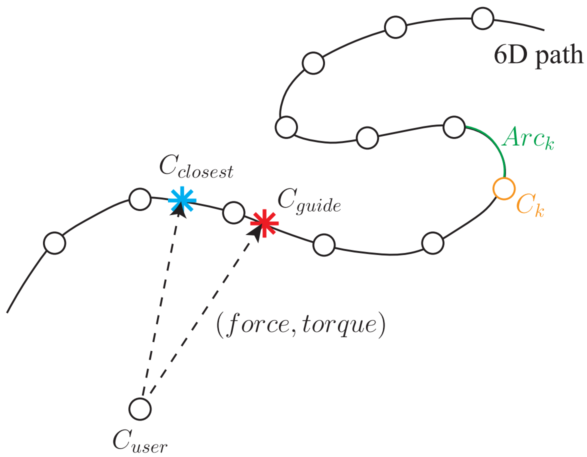


Fig. 6. Six-DoF assembly path guidance.

assume that the current position of an object operated by the user is $\mathbf{x} \in \mathbb{R}^3$ and its orientation is represented as a quaternion $\mathbf{q}$. We describe the known assembly path as a sequence of 6D configurations C_k and arcs Arc_k, whereby Arc_k interpolates between C_k and C_{k+1}.

1) Continuous Forces: At any moment of time, denote the current 6D configuration of the manipulated rigid object by C_{user}. The purpose of haptic guidance is to exert a guiding force and torque that guides the user towards the goal. Of course, the user will rarely be positioned exactly on the assembly trajectory, but will typically stray away from it by variable amounts. Given the known 6D assembly trajectory, we guide the user to a configuration that is close to, but slightly ahead, of the closest point to C_{user} on the 6D assembly trajectory. The calculation process consists of three steps, as shown in Fig. 6: (1) find the configuration $C_{closest}$ on the path that is closest to the user's current configuration C_{user}; (2) move $C_{closest}$ forward down the assembly path for a certain distance, obtaining configuration C_{guide}, so that the user is guided to make progress; and (3) calculate the guiding force and torque between C_{user} and C_{guide}.

We define the distance metric between two 6D configurations $C_1 = [\mathbf{x}_1, \mathbf{q}_1]$ and $C_2 = [\mathbf{x}_2, \mathbf{q}_2]$ as

$$D(C_1, C_2) = ||\mathbf{x}_1 - \mathbf{x}_2||^2 + \gamma^2(1 - |\cos\Omega|), \tag{1}$$

where γ is a trade-off parameter and $\cos\Omega$ is the dot product of $\mathbf{q}_1$ and $\mathbf{q}_2$. The parameter γ needs to be adjusted based on the units used for translation. In our car engine example, we set it to $\gamma = 0.5$. Given two 6D configurations C_1 and C_2, any in-between configuration, denoted as $C(t, c_1, c_2) = [X(t, x_1, x_2), Q(t, q_1, q_2)]$, can be computed via linear interpolation and spherical linear interpolation (SLERP), respectively:

$$X(t, \mathbf{x}_1, \mathbf{x}_2) = (1-t)\mathbf{x}_1 + t\mathbf{x}_2\,, \tag{2}$$

$$Q(t, \mathbf{q}_1, \mathbf{q}_2) = \frac{\sin((1-t)\Omega)}{\sin\Omega}\mathbf{q}_1 + \frac{\sin(t\Omega)}{\sin\Omega}\mathbf{q}_2. \tag{3}$$

In general, $\Omega > 0$. Otherwise, if $\mathbf{q}_1 = \mathbf{q}_2$, which implies $\sin(\Omega) = 0$, we can use the 3-DoF interpolation method as $Q(t, \mathbf{q}_1, \mathbf{q}_2) = (1-t)\mathbf{q}_1 + t\mathbf{q}_2$.

In order to guide the user from the configuration $C_{user} = [\mathbf{x}, \mathbf{q}]$ forward to C_{guide} along the path, we compute the closest on-path configuration $C_{closest}$ to the trainee's current configuration C_{user}. We give the algorithm to compute $C_{closest}$ in Appendix A. We want the user to make progress down the path, as opposed to merely being guided to the path. Given two configurations on the planned path $C_1 \in Arc_i$ and $C_2 \in Arc_j$, for $i \leq j$, we define the distance between these two configurations *along the path* as

$$L(C_1, C_2) = \sqrt{D(C_1, C_2)}, \quad \text{for} \quad i = j, \quad \text{and} \tag{4}$$

$$L(C_1, C_2) = \sqrt{D(C_1, C_{i+1})} + \sqrt{D(C_j, C_2)} + \sum_{k=i+1}^{j-1} \sqrt{D(C_k, C_{k+1})}, \quad \text{for} \quad i < j. \tag{5}$$

Therefore, we determine C_{guide} by moving $C_{closest}$ forward down the assembly path so that

$$L(C_{closest}, C_{guide}) = \ell_{max} \exp\Big(-\frac{\sqrt{D(C_{user}, C_{closeset})}}{\ell_{scale}}\Big), \tag{6}$$

where ℓ_{max} is the maximum permitted forward travel length and ℓ_{scale} is a scaling factor.

In Strategy 7 ("**Continuous guiding force**"), we use configuration $C_{guide} = [\mathbf{x}_{guide}, \mathbf{q}_{guide}]$ as the guidance configuration, and compute haptic guiding force $\mathbf{F}_{guide}$ and guiding torque $\mathbf{T}_{guide}$ as

$$\mathbf{F}_{guide} = k_{guide,f}(\mathbf{x}_{guide} - \mathbf{x}_{user}) \tag{7}$$

$$\mathbf{T}_{guide} = \frac{k_{guide,t}}{2} Scalar\Big(\frac{\mathbf{q_{guide}}}{\mathbf{q}_{user}}\Big) Vector\Big(\frac{\mathbf{q_{guide}}}{\mathbf{q}_{user}}\Big), \tag{8}$$

where $Scalar(\mathbf{q})$ and $Vector(\mathbf{q})$ denote the scalar and vector part of quaternion $\mathbf{q}$, respectively. Note that $\mathbf{T}_{guide}$ is the torque corresponding to the equivalent axis of rotation between quaternions $\mathbf{q_{guide}}$ and $\mathbf{q_{user}}$ [43]. We call this strategy "continuous" because the guiding forces and torques are displayed continuously, without any interruption. During the training, the user follows the continuous guiding force down the assembly path. As $C_{closest}$ is not always a continuous function of C_{user}, we use two rules to improve the guiding force smoothness: (a) C_{guide} should never go backwards, that is, the guidance progress cannot be reversed; and (b) we constrain the distance from C_{user} to C_{guide} to a fixed maximum value.

2) Nudging Forces: In Strategy 6 ("**Nudge force**"), we use the same process as described above to compute C_{guide}, except that we do not render the haptic guidance forces and torques continuously. Instead, we render a short guidance force and torque pulse (a "nudge"; duration is 250 ms). The nudge is applied only when the user requests it by pressing a key on the keyboard. Because the nudge force is not rendered continuously, we can permit C_{guide} to go backwards, consistent with the current user configuration. We still do not permit backward travel during each nudging pulse. We note that such nudging has not been previously adopted or evaluated as a training strategy for virtual assembly.

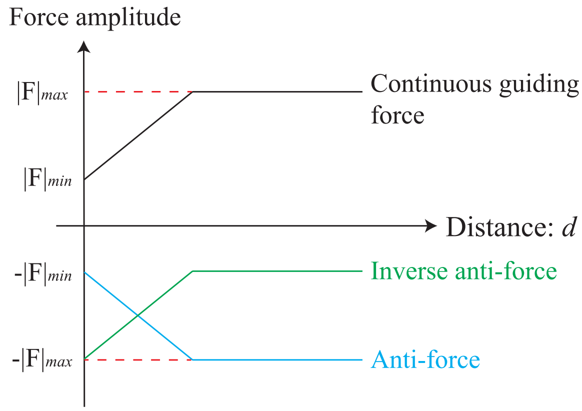


Fig. 7. Anti-force and inverse anti-force.

3) Anti-Forces: We also designed and explored "anti-force" guidance [37]–[39]. Anti-force guidance stems from the observation that by intentionally adding learning difficulties one can cause over-training, which may improve the training efficiency. We designed and tested four types of anti-forces: anti-continuous forces, anti-nudge forces, inverse anti-continuous forces, and inverse anti-nudge forces.

In the anti-continuous and anti-nudge force approaches, we set the force and torque direction to be opposite to the continuous guiding forces and torques, whereas the amplitude is set to be proportional to the user's distance d to the guide point on the path (Fig. 7). Under these two approaches, the force will be smaller if the user is closer to the correct assembly path. For inverse anti-continuous forces and inverse anti-nudge forces, we set the force amplitude to be inversely proportional to d. Therefore, the trainee is subjected to anti-guiding forces that become stronger when she or he is close to the correct path. The four anti-force strategies were supposed to train the users via "resistance". However, in our experiments, it became readily obvious that they do not actually work well. The failure occurred due to the following:

- The anti-force is designed to push the users away from the correct path. However, we observed that haptic beginners typically intuitively try to follow the force, rather than resisting it. Therefore, anti-forces can be unsafe, as users are likely to exceed the work space and cause haptic instabilities. The situation is especially difficult for anti-continuous force guidance and anti-nudge force guidance, because the force is proportional to d, and therefore the force is much larger when the user has already deviated from the assembly path.
- When teaching hand-writing and motion skills, the paths to be learned are typically two-dimensional (usually simple shapes such as arcs, circles, rectangles etc.). There are no rotations and visual occlusions, and the path is clear and easy to understand. Therefore, the users absolutely know where to go, even with disturbance forces. However, in our case, the path is complex and six-dimensional, and there are visual occlusions. The users cannot easily rely on visual observations to infer where their goal is, and how to resist the anti-force. Even if the users are experienced enough to generally understand anti-forces, they can easily become confused when subjected to anti-forces in the absence of clear visual observations.
- Unlike with teaching hand-writing and motion skills, where the goal is to strengthen the user's muscle skills to exactly repeat the path, our goal is to teach the users to complete the assembly without needing to perfectly follow any given path. We need to teach the users using a high-level pedagogical strategy that teaches them the essence of the assembly task. Anti-forces can easily be confusing because they add additional movement difficulties without offering any instruction at all.

Therefore, we did not adopt the four anti-force strategies in our user study, as they do not provide any effective assistance, and just make the user interaction less stable and potentially even unsafe.

IV. User Study

We designed a user study to test the seven training strategies described in Section III. We render our forces and torques using a 6-DoF Haption Virtuose haptic device. We used a 2.90 GHz Intel Xeon(R) CPU E5-2690 processor (32 GB RAM) and a GeForce RTX 2080 graphics card (8 GB RAM). All examples run at haptic rates (1,000 Hz).

A. Study Subjects

We recruited 56 subjects (18-30 years old) to participate in the experiment (29 male and 27 female). All subjects reported that they have no abnormalities in their upper extremities, and no vision difficulties. None of the participants had prior experience with the assembly task used in the experiment. We formed seven experiment groups in total, one for each training strategy. Each group had 56 / 7 = 8 subjects, which is consistent with other similar research in haptics [23], [44], where the number is usually 5 subjects per group (we surveyed $\sim$30 recent haptics papers). All the participants were randomly assigned to each group, except that the gender distribution was controlled to be homogeneous. All subjects were independent of each other.

B. Study Setup

In the user study, the participants were required to manipulate a handle of the haptic device to move the object in a virtual environment displayed on the screen (Fig. 8). The subjects are able to utilize three buttons on the haptic handle to rotate the camera view, zoom the camera view, and activate a virtual "clutch" to extend the workspace. The purpose of the training is for subjects to learn to assemble a starter motor into the car engine, using the strategies described in Section III. Each subject was trained using only one training strategy. We note that in several other haptic user studies (such as [44]), researchers used the "within-subjects" strategy, whereby each subject was trained and tested on several strategies. Although such a strategy could enable one to obtain more data with a smaller number of subjects, we did not use it because doing so

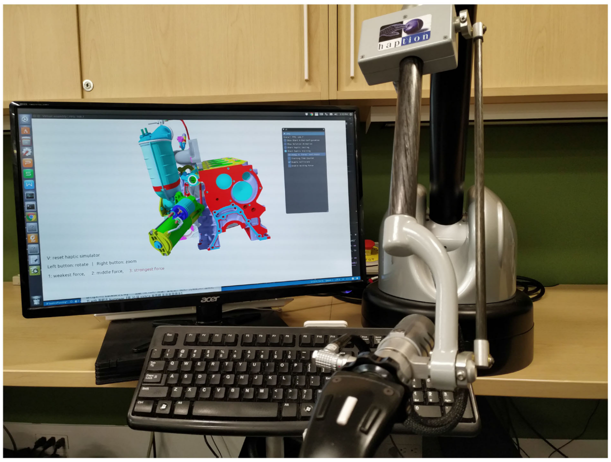


Fig. 8. User study setup.

would “pre-train” the subjects and introduce experimental noise: when training using the second, third, etc. strategy, the subject would already be partially pre-trained from previous strategies.

C. Study Stages

The user study was administered by an assistant who ensured that the subjects understand what is expected of them. The assistant was not allowed to offer any help on how to actually solve the assembly task. The assistant guided each subject through the following user study stages. The duration of the study for each subject was approximately 40-50 minutes.

1. Introduction: The assistant first briefly introduces the purpose and the contents of the user study.

2. Familiarization with the device: During virtual assembly, the subjects need to manipulate the haptic handle, control three buttons on the device, and occasionally press certain keys on the keyboard; for example, to reset the demo.Many of our users were previously not very familiar with haptics. Therefore, we designed a peg-in-hole demo for familiarization purposes (Fig. 9). This “pre-training” ensures that all participants have approximately equal haptic skills and knowledge before performing the real assembly task. The assistant informs the subject how to operate the device and how to activate the specific training assistance. Then, the subject is permitted to practice the peg-in-hole demo for several minutes. After the peg-in-hole practice, the subject is required to answer a quiz on the system usage and their specific training strategy. If the answers are not all correct, the subject needs to continue the pre-training. Only after all of their answers are correct is the subject permitted to continue to the real car engine-starter motor training.**3. Pre-training test:**The assistant first shows the subject a video of the entire assembly animation. The video is repeated 3 times. The subject is also given paper drawings of the assembly. Then, the assistant administers a pre-training test whereby the subject is asked to perform the assembly. The purpose of the pre-training test is to measure how well each specific subject performs without any training whatsoever. There is a time limit for the pre-test

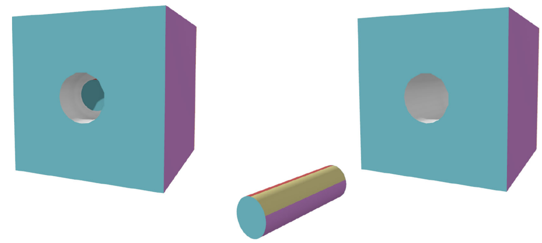


Fig. 9. Peg-in-hole assembly task.

(3 minutes) because we do not want the subjects to train themselves during the pre-training test. The assistant records the time for the subject to solve the pre-training test. If the subject is unable to solve the problem in three minutes, their pre-training test completion time is recorded as 180 seconds. After the test, the subject is asked to answer the following question: “Suppose the peg-in-hole insertion assembly is level 5 difficulty on a 1-10 scale. How hard was the engine vs. starter motor assembly?”

4. Training: Next, the subject begins to train, under her/his specified training strategy. In our pilot user study with 12 subjects [3], we found that all subjects were able to complete the assembly task at least once during the three minutes of training. Therefore, we set the training to last for four minutes. During the training, the subjects were able to trigger their specific training strategy as often as they wanted. In the groups without training assistance (Strategies 1 and 2), the subjects were told to solve the task by themselves. If the subject solved the training task before running out of time, they were encouraged to restart and repeat, in order to make full use of the training time.

5. Post-training test: After the training, the subject was tested, by having to perform the assembly three times. The assistant recorded the completion times. Finally, the subject was asked the question: “Suppose the peg-in-hole insertion assembly is level 5 difficulty on a 1-10 scale. How hard was the engine vs. starter motor assembly after training?”

V. Results

The average completion times of the pre- and post-training tests for each of the seven strategies are shown in Fig. 10, as well as their relative decrease. The subjects’ self-assessed difficulty of the engine assembly task before and after training, and their relative decrease, are also shown in Fig. 10. We obtained 56 measures of post-training test completion time from 56 participants divided into 7 groups, with 8 measures per group.

The Analysis of Variance (ANOVA) [4], [5] is a commonly used tool to statistically analyze different populations. Because we have a single categorical input variable (namely, the choice of the specific training strategy), one-way ANOVA is applicable to our case. The null hypothesis in ANOVA is that all groups are random samples from the same population, which in our case means that all the training strategies are equally effective, and any observed difference between them

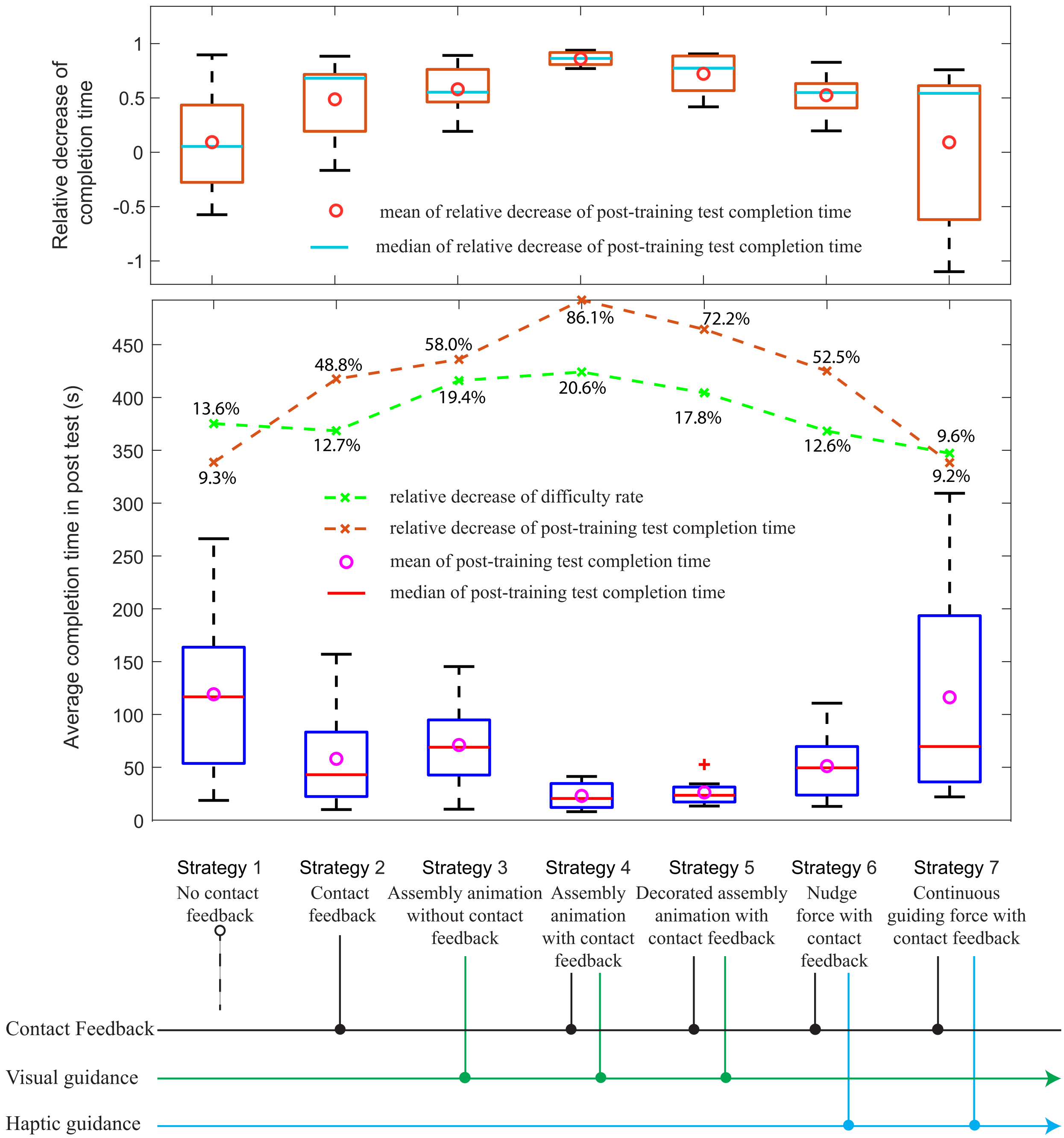


Fig. 10. Box plots of the user study results for the seven strategies.

is merely due to random chance. The p-value is the probability that, under the null hypothesis, one generates data that is as extreme or more extreme as the measured data. If the p-value is less than a threshold (a common choice is 0.05; we adopt it in our work), this justifies the rejection of the null hypothesis. Assuming ANOVA is successful, one then runs post-hoc tests to discern which pairs of strategies are statistically different from each other. We performed one-way ANOVA on our experimental results, followed by the Tukey's post-hoc pairwise comparison test [5].

The one-way ANOVA p-value of the assembly post-training test completion time was $0.0070 < 0.05$, indicating significance difference in training ability between at least two strategies. The one-way ANOVA p-value of the relative decrease of completion time is $0.0011 < 0.05$, also rejecting the null hypothesis. We then performed Tukey's post-hoc pairwise comparison test to measure the pairwise differences (Table I). Four pairs of strategies were measured to be statistically different. We note that ANOVA assumes normality of the distribution, but is generally known to be relatively robust even when this assumption is somewhat violated. We double-checked our results by also executing a non-parametric Kruskal-Wallis test [45] on our data (which is robust against non-normality), and obtained very similar results to ANOVA.

TABLE I
SIGNIFICANT PAIRWISE DIFFERENCES, MEASURED USING THE TUKEY TEST. STRATEGY 1: "NO CONTACT FEEDBACK," STRATEGY 4: "ASSEMBLY ANIMATION," STRATEGY 5: "DECORATED ASSEMBLY ANIMATION," STRATEGY 7: "CONTINUOUS GUIDING FORCE". APTTCT=ASSEMBLY POSTTRAINING TEST COMPLETION TIME, RDCT=RELATIVE DECREASE OF COMPLETION TIME

Compared strategies	p-value (APTTCT)	p-value (RDCT)
1 vs. 4	0.0344	0.0049
1 vs. 5	0.0456	0.0358
4 vs. 7	0.044	0.0048
5 vs. 7	0.0579	0.0351

A. *Interpretation of Results*

Overall, the relative decrease of the difficulty rate is strongly correlated to the relative decrease of completion time, as well as the post-training test completion time (Fig. 10). The subjects decreased the difficulty rate more if they performed better in the post-training test. Among all seven training strategies, Strategy 4 ("**Assembly animation**") and Strategy 5 ("**Decorated assembly animation**") achieved the best training results. Strategies 1 ("**No contact feedback**") and 7 ("**Continuous guiding force**") were the worst training methods.

However, counter-trends also appear in Fig. 10. In the Strategy 1 ("**No contact feedback**") and 2 ("**Contact feedback**") pair, the difference of the relative decrease of difficulty rate is opposite to the relative decrease of completion time. This means that the subjects self-assessed themselves well, while performed badly in Strategy 1. Similarly, comparing the Strategies 3 ("**Assembly animation without contact feedback**"), 4 ("**Assembly animation**") and 5 ("**Decorated assembly animation**"), the relative difficulty rate was measured to be similar, despite the subjects spending a longer time to finish the task in Strategy 3. These observations demonstrate that in Strategies 1 and 3, the subjects were likely to underestimate the assembly difficulty because of the lack of contact feedback. This reinforces a conclusion that contact feedback is crucially important for virtual assembly realism.

B. *Comparisons Within Categories*

We group our seven strategies into three categories (i.e., groups); strategies within a category are similar to each other. Although strategies within the same category share the same high-level approach, the specifics of each strategy still contribute to its efficiency. Therefore, we now compare the strategies within each category; we compare the categories to each other in Section V-C.

Category 1, "With/without contact feedback" (Strategies 1,2,3,4): For the two strategies without contact feedback (Strategy 1 and 3), the post-training test completion times both decreased to some extent. Hence, the subjects were still able to somewhat learn the assembly task even without contact feedback. Comparing Strategies 1 and 2, the subjects learned better in the strategy with the contact feedback (Strategy 2). A similar effect can be observed when comparing Strategies 3 and 4. Therefore, contact feedback is important in the training process. Moreover, the Strategy 2 and Strategy 3 had an almost equal effect on learning. This result reveals that even without visual guidance, the subjects were able to find the solution, only with the help of contact feedback. The results are comparable to assembly animation training assistance without contact feedback. Namely, the contact feedback alone can achieve a similar effect as visual guidance alone.

Category 2, "Assembly animation with/without additional decoration" (Strategies 4, 5): In Strategies 4 and 5, the p-values are larger than 0.05, denoting no significant difference between the two groups. However, the relative decrease of completion time and difficulty rate demonstrate that Strategy 4 is slightly better than Strategy 5, which is opposite to our expectation that Strategy 5 should be much better than the Strategy 4. In Strategy 4, the system directly shows the assembly animation under the subject's camera view whereas in Strategy 5, the system changes the camera view to one that we determined to be "optimal," and additionally points out the key insertion locations under those camera views. This result demonstrates that subjects are able to infer the key locations on their own, which may even strengthen the learning compared to passively receiving the same information. Our experiment demonstrate that there is no significant statistical difference in the learning success between the two strategies. Providing more training assistance does not necessarily increase the learning.

Category 3, "Guiding forces and nudges" (Strategies 6, 7): Both the relative decrease of the completion time and difficulty rate show that the Strategy 6 is better than the Strategy 7. The guiding nudge lasts a short time and has to be activated by pressing a certain key on the keyboard. During training, the subject actively learns by triggering the nudge and then continues self-exploration. In contrast, the continuous guiding force provides almost full training assistance, where the subjects just passively follow the force to complete the task. This hampers learning. Compared to Strategy 7, subjects in Strategy 6 were much more involved in learning the task, causing a higher training efficiency.

C. *Comparisons Between Categories*

Our data shows a significant difference between categories. Therefore, the guidance choice greatly affects the training efficiency. It can be seen from the box plots in Fig. 10 that the lowest bounds of completion time in post-training tests are close to each other for all the strategies. This happens because some people are inherently good at virtual assembly and can perform assembly quickly, even with little or no training. Such subjects are a minority, however, and most subjects benefit from training. Because the different strategies have different training ability, they have different variances within their subject population. Better training strategies are able to greatly improve the performance of lesser skilled subjects, thus narrowing the variance. Compare, for example, the variance of Strategies 4 and 7.

Category 1 vs. 2, "No guidance vs. Visual guidance": Strategies 4 and 5 are the best strategies among our seven investigated strategies. They have the highest relative decreases of both completion time and difficulty rate.

Moreover, our Tukey tests (Table I) measured a significant difference between Strategies 1 and 4, and Strategies 1 and 5. This demonstrates that by selecting a quality training strategy, one can outperform training without guidance, especially one without contact feedback.

Category 1 vs. 3, "No guidance vs. Haptic guidance": According to our data, Strategy 6 is slightly better than Strategy 2, due to providing assembly training assistance. However, Strategy 7 is worse than Strategy 2. During training, continuous forces guide the subjects to smoothly and successfully complete the assembly several times. However, they also produce a collision-free assembly path, preventing the subjects from making mistakes. Therefore, the subjects do not have an opportunity to learn from their mistakes. This was a frequent occurrence in Strategy 2. It is better for the subjects to actively experiment and discover the solution on their own, rather than be directly provided with the solution path. This supports our conclusion that training assistance is needed to teach difficult assembly tasks, but too much assistance hampers the learning process.

Category 2 vs. 3, "Visual guidance vs. Haptic guidance": Visual guidance Strategies 4 and 5 are more efficient than haptic guidance Strategies 6 and 7. Notably, Strategies 4 and 5 are significantly different from Strategy 7 (Table I). Our data shows that visualizations are better than guidance forces in teaching 6-DoF motion, especially rotations. Humans can easily perceive and remember visualized motions, but find it harder to memorize muscle movement, especially with rotations.

Multiple learning modalities: In our user study, we strived to isolate the individual factors combined with haptic contact, hence we evaluated, for example, the strategies of visual guidance + haptic contact (Strategy 4) and haptic guidance + haptic contact (Strategy 7). One could in principle test strategies that combine multiple learning modalities, such as, for example, visual guidance + haptic guidance + haptic contact. However, this would increase the training time as the subject would then need to be first instructed to use both visual guidance and haptic guidance modalities, and then actually actively use them during training. In our experience, when presented with multiple simultaneous learning modalities, subjects easily become confused about which specific modality they should use. When designing our experiment, we observed that most subjects naturally gravitate to just use one training modality. They would need to be consistently encouraged to use both modalities in tandem, which would cause the user study administrator to intervene into the experiment too much.

VI. Conclusion

We implemented a haptic virtual assembly training system for complex 6-DoF assembly tasks. We designed fourteen training strategies, involving contact force feedback (or lack thereof), visual guidance and haptic guidance. We also designed and conducted a user study to evaluate the efficiency of the most performant seven training strategies. Our results demonstrate that contact feedback is very important for realistic virtual assembly: when there is no contact feedback, subjects underestimate the assembly difficulty. Providing proper training assistance can substantially improve assembly performance. However, too much assistance impedes learning, due to passive participation. It is better for the subjects to actively experiment and discover the solution on their own, rather than be directly provided with the solution path. Visual guidance is better than haptic guidance at teaching complex six-dimensional motion. Humans can easily perceive and remember visualized motions, but find it harder to memorize muscle movement, especially with rotations. However, the results for our Strategy 5 demonstrate that too much visual guidance is not necessarily better. Our subjects were are able to infer key locations along the assembly path on their own just from watching solution animations (Strategy 4), and doing so strengthened their learning. When starting this research, we expected that continuous force guidance combined with haptic contact rendering (Strategy 7) will be superior to haptic contact rendering only (Strategy 2). Our data, however, shows that this is not the case: Strategy 2 is clearly better than Strategy 7, and at least equal (if not slightly better) to haptic nudging with contact feedback (Strategy 6). We think this is because haptic stimulus alone reaches diminishing returns as more and more haptic stimulus is added; it must be combined with improved visualization (Strategy 4) to improve the overall experience.

In this paper, we focused on the haptic rendering aspects of virtual assembly. We did not intend to investigate visualization at a deep level. Our results, however, demonstrate the importance of display technology. Evaluating state-of-the-art displays when combined with various haptic rendering training strategies would be interesting future work. For example, one could investigate VR immersive displays in combination with haptic training, as such displays provide improved visibility, scene perception and spatial awareness. Our work assumes that the assembly path is known, which is a typical situation in engineering. We designed our algorithms to calculate visual and haptic guidance based on a given assembly path. We did not investigate real-time path planning. Our decorated animations were designed manually by us for our specific assembly path. In the future, one could automatically identify locations on the assembly path where such instructions are needed; for example, by measuring the 6-DoF distances-to-contact along the assembly path, or measuring the local 6-DoF velocities. Our subjects interacted with the simulator by holding a haptic handle, which is a standard and common interface in haptic rendering. In the real world, assembly personnel hold complex rigid objects with two hands, with spatially varying hand skin contact pressures providing important additional cues into the assembly procedure. Incorporating such effects into our training simulator is important future work.

Appendix
Closest configuration on the 6D path

Given a 6D interpolated arc between two configurations $C_1 = [\mathbf{x}_1, \mathbf{q}_1]$ and $C_2 = [\mathbf{x}_2, \mathbf{q}_2]$, the distance between the trainee's configuration $C_{user} = [\mathbf{x}, \mathbf{q}]$ and the configuration

$C(t, C_1, C_2)$ on the arc is:

$$\begin{aligned} D(C_{user}, C(t, C_1, C_2)) &= ||\mathbf{x} - (1-t)\mathbf{x}_1 - t\mathbf{x}_2||^2 \\ &+ \gamma^2\left(1 - |\frac{\sin((1-t)\Omega)}{\sin\Omega}(\mathbf{q}\cdot\mathbf{q}_1) - \frac{\sin(t\Omega)}{\sin\Omega}(\mathbf{q}\cdot\mathbf{q}_2)|\right) \\ &= ||\mathbf{x} - \mathbf{x}_1 + (\mathbf{x}_1 - \mathbf{x}_2)\frac{\phi - \theta}{\Omega}||^2 + \gamma^2\left(1 - \frac{A|\cos\phi|}{\sin\Omega}\right), \quad (9) \end{aligned}$$

where

$$A = \sqrt{((\mathbf{q}\cdot\mathbf{q}_2) - (\mathbf{q}\cdot\mathbf{q}_1)\cos\Omega)^2 + ((\mathbf{q}\cdot\mathbf{q}_1)\sin\Omega)^2}, \quad (10)$$

$$\cos\theta = \big((\mathbf{q}\cdot\mathbf{q}_1)\sin\Omega\big)/A, \quad (11)$$

$$\sin\theta = \big((\mathbf{q}\cdot\mathbf{q}_1)\cos\Omega - (\mathbf{q}\cdot\mathbf{q}_2)\big)/A, \quad (12)$$

$$\phi = t\Omega + \theta. \quad (13)$$

Because quaternions $\mathbf{q}$ and $-\mathbf{q}$ represent the same configuration, we can always choose the proper $\mathbf{q}$ so that $\mathbf{q}\cdot\mathbf{q}_1 \geq 0$, resulting in $\theta \in [-\pi/2, \pi/2]$.

Because the distance metric is continuous, the configuration on the arc $C(t, C_1, C_2)$ that has the shortest distance to C_{user} will either be one of the two ending configurations of the arc, or one where the derivative of the distance metric function is zero. Hence, we can further simplify the derivative of the distance function to

$$\begin{aligned} &\frac{dD(C_{user}, C(t, C_1, C_2))}{dt} \\ &= \frac{\gamma^2\Omega A}{\sin\Omega}\Big(-\frac{d|\cos\phi|}{d\phi} + \frac{2\sin\Omega}{\gamma^2\Omega^2\ A}||\mathbf{x}_1 - \mathbf{x}_2||^2(\phi - \theta) \\ &+ \frac{2\sin\Omega}{\gamma^2\Omega A}(\mathbf{x}_1 - \mathbf{x}_2)\cdot(\mathbf{x} - \mathbf{x}_1)\Big) \\ &= \begin{cases} \frac{\gamma^2\Omega A}{\sin\Omega}(\sin\phi - k\phi - b), & \phi \in (-\pi/2, \pi/2) \\ \frac{\gamma^2\Omega A}{\sin\Omega}(-\sin\phi - k\phi - b), & \phi \in (\pi/2, \pi), \end{cases} \quad (14) \end{aligned}$$

where

$$k = -\frac{2\sin\Omega}{\gamma^2\Omega^2\ A}||\mathbf{x}_1 - \mathbf{x}_2||^2, \quad (15)$$

$$b = -\frac{2\sin\Omega}{\gamma^2\Omega A}(\mathbf{x}_1 - \mathbf{x}_2)\cdot\left(\frac{\theta}{\Omega}(\mathbf{x}_1 - \mathbf{x}_2) + \mathbf{x} - \mathbf{x}_1\right). \quad (16)$$

From the above equation, it follows that the zero-derivative configuration can be obtained by calculating the intersections between a $\sin$ function and a negative-sloped straight line with $\phi \in (\theta, \Omega + \theta)$. Because $k < 0$, the function $k\phi + b$ is strictly decreasing while the $\sin$ function is strictly increasing. Therefore, if $\phi \in (-\pi/2, \pi/2)$, there is at most one solution of the equation $\sin\phi = k\phi + b$ (see Fig. 11). We obtain the solution as follows. First, we need to determine if there is zero or one solution of the equation. We compute the values at the two ending points of $k\phi + b$ and check if they are above or below the $\sin$ function. If yes, there is no intersection. If no, we use binary search to find the solution. The case where $\phi \in (\pi/2, \pi)$ can be treated in the same way. We find the closest location on the entire path by checking each path arc individually and taking the minimum. This did not pose a computational problem in our application as the path only consists of approximately 100 segments (which can be processed in parallel).

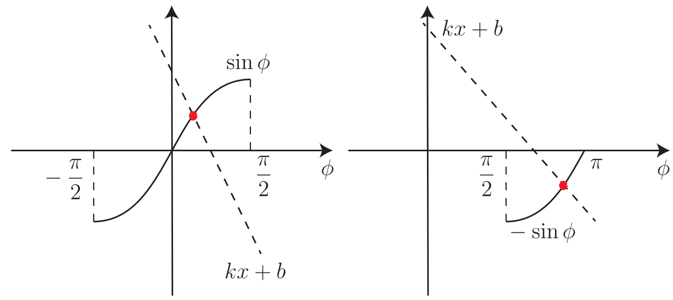


Fig. 11. The intersection point in Equation (14). Left and right subfigures correspond to the two cases of Equation (14).

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